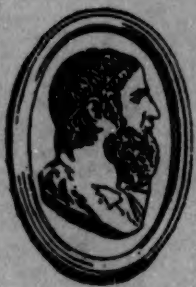
THE MATHEMATICS TEACHER



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John Wallis

THE MATHEMATICS TEACHER

Volume XXV Number 7

Edited by William David Reeve

Getting the Most Out of Objective Tests in Mathematics

By Fletcher Durell and Thomas J. Durell
Belleplain, New Jersey

In recent years the so-called objective tests have come into wide use and have enjoyed a well deserved popularity. In particular they form a useful auxiliary to what is often termed the new education; that is, those ideas and methods which stress the pupil's self-activity and a more or less democratic form of school administration and classroom procedure.

So valuable have the new type tests already proved themselves in this connection that it seems worthwhile to use every method of study and experiment to get the utmost value out of them. It is the object of the discussion which follows to try to make some contribution along this line.

In this discussion we limit ourselves to the four types commonly known as *completion, true-false, multiple choice,* and *matching* tests.

Completion Tests

The so-called completion tests are especially well fitted to cultivate in the pupil in simple direct fashion, the values which we have just mentioned as characteristic of the new education.

As a basis for the first part of the discussion let us consider the following small group of somewhat related statements of the completion type.

Ex. 1. A quadrilateral is a polygon having .... sides.

Ex. 2. A quadrilateral with .... equal sides and .... equal angles is called a square.

Ex. 3. A quadrilateral having all of its sides and angles equal is a ....

Ex. 4. A square is a quadrilateral having all of its sides .... and all of its angles ....

When the pupil completes any one of these statements there is a certain amount of co-operation between the pupil and those giving him instruction. Thus in Ex. 1, seven words are supplied by the instructor and the pupil supplies one word. This makes for a friendly atmosphere and a sense of swift accomplishment because of joint action and mutual helpfulness.

This sense of co-operative activity is also intensified in a somewhat subtle way by the declarative form in which completion examples are expressed, as distinguished from the interrogative and imperative forms. For instance instead of the question "What is a triangle?" or the command "Give a definition of a triangle" we have the confident statement, "A triangle is a polygon with ...... sides."

Here the declarative form seems to imply that there is no doubt about the pupil's ability to fill the missing blank; that, in fact it will be natural and easy and a matter of course for him to do this. The expression of such confidence in him gives him confidence in himself and thus tends to arouse all of his powers into activity.

The old method of learning definitions and rules by pure rote memory and a parrot-like reproduction of them not only was very dull and uninteresting to pupils but also often seemed to have a kind of stupefying effect upon them so that they really did not understand what they were talking about. The use of completion examples in this connection, not only makes most of this drudgery unnecessary but gives a vivid and many-sided grasp of the meaning of mathematical terms. It converts the process of acquiring this knowledge into something with the spice of a game; a kind of abbreviated but super cross word puzzle. The pleasure thus experienced arouses still further the pupil's spontaneous activity.

All agree that one of the highest forms of self-activity which education should develop in pupils is the power to think or reason. Com-

pletion and other new type tests are well fitted to arouse this faculty in simple initial ways.

Under traditional methods of education it has been a question frequently debated whether, even when pupils have learned to carry on processes of reasoning in mathematical fields, such powers transfer to other departments of life. No doubt frequently they do not thus transfer. One reason for this may be that a formal demonstration, for instance, may be too large a unit and one of too special and technical a character to make it as a process readily usable in other situations.

It is said of Abraham Lincoln that when in adult life he first heard the word "demonstrate" he questioned a friend closely as to the exact meaning of the term. His friend advised him to procure a copy of Euclid and study its contents. This advice Lincoln followed. There can be little question that the knowledge and appreciation of the power to reason which he thus gained was of life-long benefit to him in many ways. His Gettysburg speech, for instance, among its other prominent characteristics, has the approximate length, form, and other qualities of a geometric demonstration. It may be that Lincoln was helped in getting such values from this study because in teaching himself, he would naturally break the logical whole up into individual steps and carefully test and examine each in the spirit of his legal training.

Be that as it may as stated above, instead of large, self-contained, closely knit special logical wholes, completion and similar tests supply small unit exercise steps in making inferences, with many pleasurable and stimulating associations, and are thus especially adapted to instill a spirit of deductive inference, and to enable the pupil to take first steps in this field.

Thus, in Ex. 1. given in the group above, the pupil must use some thought power but it is only the small amount required in avoiding such words as "several" or "equal" in filling in the blank. In the three examples which follow, there is a gradual increase in the amount of intelligent attention called for.

Other completion examples calling for greater powers of deduction are the following:

Ex. 5. The sum of the angles of a hexagon is .... right angles.

Ex. 6. A quadrilateral which contains two pairs of equal angles must be a .... or an ....

Ex. 7. Two similar radicals may differ in .... and ....

It is to be noted also that examples like No. 6 above contain the beginnings of research, a somewhat higher form of self activity than those thus far discussed. Other examples of the same sort are the following, to be used in connection with the ordinary study of solid geometry.

Ex. 8. A solid figure bounded by one plane and one curved surface may be a .... or a ....

Ex. 9. A solid figure bounded by two planes and one curved surface may be .... (Let the pupil complete the example.)

Such examples are also adapted to arouse class discussions and co-operative research on the part of groups of pupils.

In completion examples it is customary to have each blank or dash stand for a single word. A more difficult type of completion example can be framed by having a blank sometimes represent a group of words as a phrase or even a sentence.

Ex. 10. A solid bounded by two planes and one curved surface may be a ...., a ...., or a ....

Problems of this kind evidently call for self-activity, and often a higher sort of reasoning.

True-False Tests

The values which have been mentioned in connection with completion examples are also obtainable from the next class of objective tests which we take up, *viz.*, true-false tests. In addition this second type have other values peculiar to themselves though they are also beset by certain characteristic dangers or possible drawbacks. We will first of all briefly consider this matter of pitfalls.

The chief of these is the dangers that the pupil forms the habit of guessing at answers.

In case he has no clear light as to whether a given statement is true or false, often a pupil will base his answers on some vague probability, not on logic or understanding. This is especially true in the case of the more difficult true-false statements, which are, of course, the more important.

But by care on the part of the teacher and textbook writer this liability may be converted into an asset. The remedy is that the pupil be required to rewrite in correct form every statement which he has marked as false. All examples should be so selected and worded

that an intelligent understanding of the matter is required in the rewriting in the true form. Each example should be so framed that the mere insertion (or removal) of the word "not" will not constitute a satisfactory revision of a false statement. Take this statement:

Ex. 11. The sum of the angles of a triangle is four right angles.

To mark this false and rewrite in the form, "The sum of the angles of a triangle is not four right angles," would not be satisfactory. The proper revision is of course:

The sum of the angles of a triangle is two right angles.

This practice of rewriting principles and facts fully and accurately has an additional value as a training in English expression.

Another criticism of the true-false test is that it is bad pedagogy to place an actually false statement in print before the pupil because it may be impressed on his mind with a kind of permanence. The answer is that in after life the pupil will meet many false statements in print and must learn to deal with them. If he forms the habit of analyzing these and reconstructing them in a true form, he will be acquiring a power which will be very helpful in later years in dealing with printed errors of all kinds.

When treated thus, true-false tests will be found to be a very satisfactory way of obtaining the same useful educational results already described in the discussion of completion tests, such as arousing the interest and self-activity of the pupil, his sense of co-operation with the teacher and the textbook, and his power to reason.

Simple true-false examples where some intelligent thought power is required are the following:

Ex. 12. If one angle of a triangle is obtuse, each of the other angles must be acute.

Ex. 13. The sum of the angles of a pentagon is 4 straight angles.

An advantage of such examples is that in them may be expressed every principal kind of false logic to which pupils are prone.

There is the danger also of making true-false examples too difficult, with flaws too hidden and subtle. It requires care and study to devise statements which contain just the right amount of challenge, and are neither too difficult nor too easy.

The true-false test has another useful function which is peculiar to itself. This is the aggressive, comprehensive, and constructive way

in which it treats the negative side of knowledge. It is often more important to understand what a principle is not than the more positive aspect of the matter. In the same way, it is frequently as necessary to know the negative side of a person, his limitations and defects, as to realize his positive powers. Taken by itself this shadow side of life is not so attractive as the other, and in school it is too often unpleasantly associated with penalties and disasters. For this reason, too little attention has been paid to it in education. If it is treated in direct and varied co-operation with the positive side, and aggressively associated with achievement, the study of this negative phase may be made both systematic and agreeable. This is what can be done in true-false tests, and this is one of their distinct and unique contributions to education.

Not infrequently statements are included in a true-false test which are sometimes true and sometimes false. For instance:

Ex. 14. A quadrilateral with 4 equal sides is a square.

Ex. 15. A solid bounded by two parallel planes and one curved surface is a cylinder.

In the directions for working such examples, the pupil is told to consider such statements as false.

This custom of having the pupil mark as absolutely and always false a statement which is sometimes true is apt to lead to quibbling and juggling with words, and is both pernicious and unnecessary. By the use of such words as "every" and "all" the ambiguity can readily be removed from such examples. Thus, instead of Ex. 14. above, we have:

Ex. 16. Every quadrilateral with 4 equal sides is a square.

T-F-S Tests

However, statements like Ex. 14 and Ex. 15 may be made of distinct educational value by including them in a special type of test. Such a test would consist of three, instead of two, kinds of statements, (1) those absolutely true, (2) those absolutely false, (3) those sometimes true and sometimes false. The pupil would be asked to mark each statement either T (for true), F (for false), or S (for sometimes true). Each statement which the pupil marks S, he should rewrite or modify to show under what conditions it is true. So Ex. 14 might be rewritten as follows:

Ex. 17. A quadrilateral with 4 equal sides and 4 equal angles is a square.

This proposed true-false species of true-false tests will probably call for an unusually high degree of power to analyze, discriminate, and often to reason. The statement in words of the facts derived will also provide an exceptional training in the use of clear, accurate, and logical English.

Thus again a liability will be turned into a valuable asset.

Multiple Choice Tests

From one point of view a multiple choice statement is a special form of the completion type. This fact may be illustrated by the following example:

Ex. 18. Select the correct answer from the list of possible results in: $0.05 \times 84 = 420$, *42, 0.042, 4.2 or 0.0042.*

In this example what the pupil is essentially required to do is to complete the statement, 0.05 x 84= ...., the blank to be filled by a number selected from a given set of alternatives. Hence the values which come from the use of completion tests occur also in multiple choice drills, and may be considerably enhanced.

From another point of view, the multiple choice type may be viewed as a special form of the true-false test. For in Ex. 18 there really have been combined five statements, four of which are false and one true. Hence the values connected with the true-false test are also present in the multiple choice variety.

If the multiple chioce test is fully developed, the values which it shares with the completion and true-false types may be greatly increased. It is customary in the list of possible answers to have only one correct result. The disadvantage of this is that if the pupil knows this to be the case, as soon as he has come to a correct answer the remaining items in the list have no further interest for him.

A much better method is to include in the list of alternatives two or even more correct answers, as in the following example:

Ex. 19. $\sqrt{12} = 4\sqrt{3},\ 2\sqrt{3},\ 3\sqrt{4},\ 6 \div \sqrt{3},\ 2/3\sqrt{27}$

Obviously, when the pupil knows that there may be more than one correct answer and that he is supposed to select every correct answer, he will be on the alert till he has tried the last item in the list, and the solution of the problems will become a many-sided game.

Besides the values which the multiple choice type has in common with the two preceding types, it has other values peculiar to itself. The most important of these is the means afforded of showing the equivalence of different primal mathematical concepts, causing them to throw light on each other and to aid each other in various ways. In short, the multiple choice test furnishes an opportunity for teaching what we have elsewhere termed co-operative mathematics.

Consider this example:

Ex. 20. $\frac{1}{4}=.25,\ 2.5\%,\ 1:4,\ 25\%,\ .025,\ 4\div 1,\ 1\div 4,\ \frac{1}{4}\%$.

We have here a co-operative correlation between the topics of fractions, decimals, per cents, and ratio.

This method can be used in a progressive, cumulative way. At a later stage in the pupil's mathematical development, after he has studied the general topic of exponents in algebra, items like the following can be annexed to the list of alternative answers to Ex. 20:

$$4^{-1},\ 1^{-4},\ 4^{0},\ 2^{-2}$$

By thus providing a many-sided facile grasp of the co-operative relation of the primal concepts of mathematics in an elementary way, the use of the idea of co-operative mathematics in connection with more difficult topics and larger units will be made easier and more natural. For instance:

Ex. 21. If $C=2\pi r$, *state which of the following is true:*

$r=\dfrac{2\pi}{C}$; $r=C\div 2\pi$; $\pi=\dfrac{C}{2r}$; $r=C-2\pi$; $r=2C\div\pi$; $C:r=1:2\pi$; $C:r=2\pi:1$; $C:r=\pi:\frac{1}{2}$; $\pi=\dfrac{2r}{C}$; *C varies as r; r varies as C; if r is doubled C is multipled by 4; etc.*

This example affords a correlation of equations, formulas, proportion, variation, and dependence.

It also illustrates an additional aspect of the idea of reasoning, *viz.:* that of extracting the maximum number of inferences, i.e., the utmost amount of information from a given statement or relation.

All the multiple choice examples which have been given here also make clear the idea that they afford an excellent opportunity of listing the more prevalent and fundamental errors and recurring to them repeatedly as an aid in their cure.

Matching Tests

The following example in matching will be useful as a basis of our discussion of this type of problem.

Ex. 22. In the following table state which items in Column II are true for each item in Column I:

Area of	*Formula*
(1) Triangle	*(a)* $A=s^2$
(2) Square	*(b)* $A=\frac{1}{2}h(b_1+b_2)$
(3) Rectangle	*(c)* $A=bh$
(4) Parallelogram	*(d)* $A=\pi r^2$
(5) Trapezoid	*(e)* $A=\frac{1}{2}bh$
(6) Circle	*(f)* $A=\frac{1}{4}\pi d^2$

If this example is carefully analyzed, it will be realized that a matching test is a composite of several examples of the multiple choice type. For all the items in Column II constitute a list of possible correct answers for each item in Column I.

Hence the values which we have listed as characteristic of the multiple choice variety are true also in a manifold way for the matching type.

In the matching test also the values are enhanced by having an occasional item in one of the columns correspond to two or more items in the other column. Thus, in Ex. 22, item (2) in the lefthand column corresponds to both (a) and (c) in the righthand column. Item (6) also matches two items in Column II. Inversely, (c) in Column II is true for three items in Column I.

This multiplicity of correspondences compels more alertness on the part of the pupil and a greater exercise of his reasoning powers.

Combination of the Four Types

Thus far we have discussed methods of getting as large values as possible out of each of the four types of objective tests considered separately. It remains to consider briefly ways of securing full values out of these tests when used together in wise co-operation. For all four types may be made to work together to produce other worthwhile learnings in addition to those already mentioned as characteristic of the individual types.

For instance, by taking advantage of the pleasure which pupils experience in working objective tests, it is possible so to utilize these various tests as to make them a valuable aid in keeping up a con-

tinuous interest in the subject. At any point where the study begins to lose its direct appeal, perhaps because of some necessarily extended, theoretic discussion, it is possible occasionally to introduce one of these quick-moving, self-active exercises. So by proper distribution and combination of exercises of the different kinds under discussion we can keep up a kind of continuous pleasure while at the same time the development of the subject matter is systematic and logical in its structure.

Again, these exercises have a particularly useful function with respect to reviews. The element of mere repetition characteristic of most reviews tends to make reviews tedious and uninteresting to most pupils. Objective tests remedy this by their comparative brevity and by showing aspects and relations among the various items recalled.

These new type reviews may also be freshened up by introducing into them items of subject matter for which there was not room in the original development.

The many-sided grasp of the primal elements of a subject treated by such new type reviews also tends to make the more elaborate reviews of the advanced technical parts of the subject largely unnecessary, or at last to make it possible greatly to abbreviate such reviews. Co-operative mathematics is thus made easier and more natural in the longer and more elaborate processes.

These tests are also especially adapted to develop certain forms of original power and creativeness on the part of the pupil. For example:

Ex. 23. Make up three completion examples covering the definition of a triangle.

The amount of self-activity required to answer a question like this is so small that almost any pupil can do the work required and thus make a start in this higher kind of creativeness.

A later development would be to have a class discussion on the characteristics of a good completion example, and on the basis of these standards developed by the class have the pupils select the best examples handed in by the class and make them up into a well-balanced test. The same procedure applies also, of course, to true-false, multiple choice, and matching tests.

The way will also be open to devise special variations of objective tests which are in effect new types of tests but based on the original forms. Take the following examples:

Ex. 24. Find the value of $(87+90+92+88+89+93)\div 6$. *Express this as a problem concerning the average grade of a pupil.*

Ex. 25. Find the value of $(200-50)\div 5$. *Express this as a problem concerning an article bought on the installment plan with a down payment of $50.00.*

In such examples we have a kind of inverse completion. The figures and processes are given, and the pupil is asked to supply the language or story. Such examples are especially useful as an aid in training pupils to grasp as a unified whole all the processes involved in a given problem, which is one of the most worth-while powers which a pupil can possess.

When we consider how large a part of our life activities and processes consists in taking certain data and by study and experiment filling in certain gaps in relation to them; how much also consists in discrimination between the true and the false, and how much in matching equivalents, we get some idea of the vast field open for the application of the powers trained by the study of these objective tests, especially if the spirit of their higher forms is developed.

It seems that we are only at the beginning of grasping the values which may be derived from further investigation and experiment with the types of exercises which we have been discussing in this article.

A Study of Certain Mathematical Abilities in High School Physics*

By William Ray Carter
University of Missouri, Columbia, Missouri

IV. Description of Subjects

Geographical Distribution of Schools

The students included in this study were members of the physics classes of thirteen high schools of Missouri during the school year of 1930-31. In selecting the schools which were to co-operate in this study, the Missouri State High School Directory for 1929-30 was used to make a list of the first class high schools offering physics during that year. Letters outlining in detail the nature of the proposed study and asking co-operation in the project were sent to the principals of these schools. The thirteen schools included in this study were the ones from which came the first offers of co-operation. In order to check the results from the Carter test, certain data from the physics classes of the Cleveland High School of St. Louis were used. Data from this school, however, were not used in the main body of this study. There were twenty separate physics classes in the thirteen schools included in this study. The locations of the co-operating schools are shown in Table I.

Size of Schools and of Classes

The enrollment in the schools co-operating in this study ranged from 170 to 1500, while the enrollments in the physics classes of these schools ranged from a single section of seven students in one school to four sections with a total of ninety-eight students in a larger school. The total enrollment in the physics classes of the thirteen schools was 477 students. Complete data on all tests were obtained for 404 students, or 84.7 per cent of the total number. This study includes only the students for whom complete returns were

* This is the second installment of an extensive study by Mr. Carter. The first installment appeared in the October issue of The Mathematics Teacher.—The Editor.

obtained. The names of co-operating schools with the number of physics students in each and the number of students in each for whom complete data were obtained are found in Table I.

TABLE I

The Enrollment in the Physics Classes of the Schools Co-operating in This Study and the Number of Students in Each for Whom Complete Data Were Obtained

School	Enrollment	Complete Data
California (Mo.)	11	11
Clinton	27	19
De Soto	25	14
Farmington	27	25
Gallatin	14	14
Independence	98	88
Jefferson City	22	15
Marshall	24	24
Moberly	38	35
Nevada	62	49
Normandy	7	7
St. Charles	24	22
Southwest (Kansas City)	98	81

Classification of Students

In the schools included in this study the membership in the physics classes was largely made up of seniors, there being 271 seniors, 131 juniors, and two sophomores. Of the entire group, 67.1 per cent were seniors, 32.4 per cent were juniors, and 0.5 per cent were sophomores. The distribution of subjects according to grade classification is given in Table II.

TABLE II

A Distribution of Subjects According to Grade Classification and Sex

	Seniors	Juniors	Sophomores	Total
Boys	177	95	2	274
Girls	94	36	0	130
Total	271	131	2	404
Per cent	67.1	32.4	0.5	100.0

A reference to Table II shows that the ratio of seniors to others was almost exactly two to one. Two schools admitted only seniors to the physics classes, while the remainder admitted either juniors or seniors.

Sex of Subjects

The boys considerably outnumbered the girls in the classes studied. Of the total of 404 students, 274 were boys and 130 were girls. The ratio of boys to girls in the classes under consideration was, therefore, about two to one. One physics class of twenty-four students was evenly divided as to sex. In another school there were ninety-eight physics students, only eight of whom were girls. Table II summarizes the main facts in this respect.

Chronological Age Distribution of Subjects

The median chronological age of the 404 students included in this study was seventeen years two months. The range of ages for these students was from thirteen years ten months to twenty-two years ten months. Leaving out the two most extreme cases, the range of ages was from fourteen years two months to twenty years nine months. The middle fifty per cent ranged in age from sixteen years six months to seventeen years nine months. Chronological ages are reported in this study as of the time of the administration of the intelligence tests in November and December, 1930. The foregoing discussion is based upon a distribution in terms of a one month step-interval. Table III gives the distribution of chronological ages of the subjects employed in this study.

Previous Training of Subjects in High School Mathematics

The average number of semesters of high school mathematics taken before the school year of 1930-31 by the students included in this study was 4.8. This has reference to all mathematics taken beyond the eighth grade regardless of the type of school organization found in the co-operating schools. Approximately 10 per cent of the group had had only 2 semesters of high school mathematics, and about 37 per cent had had 4 semesters. Nearly 50 per cent of the group had had more than 4 semesters of mathematics, and more than 15 per cent had had 7 semesters or more. Table IV summarizes the facts regarding the previous training in mathematics for the students employed in this study.

Further reference to the facts presented in Table IV are made in Part VI and in Table XVI in connection with a discussion of the relation between mathematics training and performance on the Carter test.

TABLE III

Chronological Age Distribution of Subjects

Ages	Boys	Girls	Total
22½*	1	0	1
22	0	0	0
21½	0	0	0
21	0	0	0
20½	1	0	1
20	0	1	1
19½	5	1	6
19	5	2	7
18½	12	7	19
18	26	16	42
17½	38	24	62
17	58	24	82
16½	49	29	78
16	56	13	69
15½	13	5	18
15	9	7	16
14½	0	0	0
14	0	1	1
13½	1	0	1
Number	274	130	404
Median†	17–1	17–4	17–2
Q1†	16–5	16–7	16–6
Q3†	17–8	17–11	17–9

* The 22½ means from 22 years 6 months to 22 years 11 months plus. In other words, the figures given here are the lower limits of the respective steps.

† Taken from a distribution with a step-interval of one month. The figures are practically the same for the interval used here.

TABLE IV

Previous Training in Mathematics by Semesters Beyond Grade Eight for the Subjects Included in This Study

Semesters of Training	Number of Students	Per Cent of Group
1	0	0.0
2	40	10.0
3	16	4.0
4	148	36.7
5	58	14.4
6	79	19.6
7	49	12.1
8	12	3.0
9	2	0.5
Total	404	100.3
Mean Number of Semesters	4.8	

Table reads: There were no students with but one semester of training in high school mathematics; 40 students, or 10 per cent of the group, had had only two semesters of mathematics training, etc.

Previous Training of the Subjects in High School Science

Including all courses taken before September, 1930, the average number of semesters of work in high school science for this group was 2.3. Twenty-eight per cent of the group reported that they had had no high school science before beginning the work in physics. Three and seven-tenths per cent had had one semester of science, and 35.4 per cent had had only two semesters of science. A total of 67.1 per cent had had two semesters or less of science. The range in amount of science training was from no training to eight semesters. These facts are summarized in Table V.

TABLE V

Previous Training in Science by Semesters Beyond Grade Eight for the Subjects Included in This Study

Semesters of Training	Number of Students	Per Cent of Group
0	113	28.0
1	15	3.7
2	143	35.4
3	8	2.0
4	81	20.0
5	6	1.5
6	27	6.7
7	3	0.7
8	8	2.0
Total	404	100.0
Mean Number of Semesters	2.3	

Table reads: Of the 404 students included in this study, 113, or 28 per cent, had had no high school science training; etc.

Intelligence of Subjects

The median intelligence quotient for the members of this group on the Otis Self-Administering Tests of Mental Ability, Higher Examination, Form B, is 105.9. The middle 50 per cent of the group are between 96.4 and 113.9. The range of intelligence quotients for the whole group is from 70 to 130. The median intelligence quotient for this group is higher than that for the general high school population, which is to be expected since these students were juniors and seniors in an elective course. Table VI, which is found in Part VI, summarizes the facts for this group with respect to intelligence.

Similar results were obtained by Hurd[1] in 1925-26 in connection

[1] Hurd, A. W., "Suggestions on the Evaluation of Teaching Procedures in High

with a study involving the administration of intelligence tests to 83 physics students in three classes. He reports a median intelligence quotient of 108.7 and a range of 91 to 127 for his group. A comparison of results from the intelligence tests for these two groups gives some evidence that the group of students included in this study are fairly representative with respect to intelligence. The differences between the two groups are probably not any greater than would be expected when we take into consideration the differences in the numbers included in the respective groups.

A more complete description of the results from the intelligence test for the group included in this study is given in Part VI.

Reading Ability of Subjects

In reading ability, as measured by the Nelson-Denny Reading Test for Colleges and Senior High Schools, the group of subjects employed in this study is above average, the median for the group being 70.5 as compared to a norm of 57 for high school seniors. In other words, the median for this group is slightly above the seventy-fifth percentile for seniors, according to the author's norms. The middle 50 per cent are between 53.0 and 86.4. The main facts regarding the reading ability of the group are summarized in Table IX, which is found in Part VI in connection with a more complete description of the results from the reading test.

General Conclusions Regarding the Nature of the Subjects

This chapter presents certain facts regarding the nature of the groups of students which were used as subjects for this study. It has been shown that they were fairly representative with respect to the size of schools and of the communities in which these schools are located. The facts presented in this chapter also seem to justify the conclusion that the subjects employed in this study were fairly representative with respect to grade classification, ratio between boys and girls, chronological age distribution, intelligence, reading ability, and previous training in high school science and mathematics. It is believed that results obtained in this study, therefore, are typical of those that would be found in any group of about the same number.

School Physics." *School Science and Mathematics,* 1927, Vol. XXVII. pp. 520-526.

V. The Administration of the Tests

The Nature of the Tests

The five tests used in this study have already been described in Part III. All of these tests were standardized with the exception of the Carter Test for Mathematical Concepts in High School Physics, which is the research test developed in connection with this study. This latter test had been partly standardized as the result of the preliminary administration during the spring of 1930. All of the tests were objective as to directions for administering, type of response to be made by the pupil, and method of scoring.

Only two of the five tests, the Otis Self-Administering Test of Mental Ability, Higher Examination, Form B, and the Nelson-Denny Reading Test for Colleges and Senior High Schools, were timed tests as used in this study. The Carter Test for Mathematical Concepts in High School Physics and the Butler Test for Mathematical Concepts had no time limits, since it was desired to make these tests power tests rather than speed tests. The Kilzer-Kirby Inventory Test for the Mathematics Needed in High School Physics was supposed to be a timed test with a time limit of 40 minutes, but in order to make the results from this latter test more comparable with those from the Carter and the Butler tests, no time limit was set in the administration of this test.

Administration by Regular Teachers

Because of the length of time required for the administration of the series of tests, the varying length of class periods, and the expressed desires of certain high school principals and physics teachers, most of the tests were administered by the teacher in charge of the physics class in each co-operating school. This was considered to be a desirable procedure since it made for more normal testing conditions. Certain exceptions were made to this procedure, however. In order to become more familiar with certain problems concerned with the administration of these tests, the writer administered the complete series of tests to approximately 150 students in three high schools.[1] In addition to this, the Carter test was administered by the writer in one large high school in which there were 98 physics students.[2]

[1] These high schools were at Moberly, Mexico, and Independence.
[2] The Southwest High School of Kansas City, Missouri.

Standardized Procedures

A complete set of directions was prepared for the use of teachers who were to give the tests. This set of directions included a mimeographed sheet of general directions describing certain procedures to be followed in all tests and emphasizing the importance of a careful observance of the directions for each test in order that results might be more comparable from group to group. There was also included for each test a mimeographed sheet of directions worded exactly the same as the standardized directions published by the authors and emphasizing again the necessity for saying exactly the things outlined in the directions for that test.

Time of Administration

Since the series of tests required from three to five class periods for giving, it was found necessary to give them at intervals of one to two weeks in each school in order to interfere as little as possible with the ordinary routine of the school and in order to keep the pupils working at as high a level as possible. The tests administered by the writer were given during the latter part of November, 1930. The other tests were administered by the respective teachers during the months of December and January and the first half of February. In most cases the interval between the first test and the last in any given school was about five weeks and in no case was it more than nine weeks.

Other Data Obtained

Spaces were provided on the various tests in which the student was asked to record his age, in terms of years, months and days; his school grade; the number of semesters of training in high school mathematics in grades nine to twelve; the number of semesters of training in high school science, other than physics, in grades nine to twelve; and whether he was in the first or second semester of physics.

Conclusion

It was felt that the procedure described in the foregoing paragraphs was sufficiently objective and standardized to yield results which would be comparable from school to school. As tests were sent in, distributions were made for each group on each test in order to check, as far as possible, against any irregularities in the administration of any test. In all cases included in this study it was found

that no significant difference existed between any one of these schools and the ones in which the tests were given by the writer. Certain differences in range of scores were to be expected according to the size of the school system and the number of students enrolled in the class. It is believed that the data from the various schools were reasonably accurate and reliable.

VI. Results Obtained

Introduction

In Part I of this study the origin of the problem is discussed and certain related studies in high school physics and mathematics are summarized and criticized. The second part includes the statement of the problem and its limitations with a brief summary of procedures necessary to a realization of the purposes of the study. Certain problems and techniques involved in the measurement of mathematical abilities in high school physics are outlined and discussed, in part, in Part III, which also includes a description of the research test developed in this study and a description of the other research instruments used. A description of necessary procedures in the selection of a representative group of subjects and in the administration of the tests is given in Parts IV and V.

This section presents the results obtained from each of the five tests used in this study together with a distribution of teachers' marks in physics for the first semester. These results are stated in terms of distributions of scores, central tendencies of performance on each test and dispersions of scores. Certain comments as to the nature of the performance on each test are given in this section, but as far as possible interpretations of results and detailed conclusions are reserved for Part VII, in which these results are given further statistical treatment in order to make possible more reliable conclusions. The results given in Part VI are discussed under six main divisions:

1. Results from the Otis Self-Administering Tests of Mental Ability, Higher Examination, Form B.
2. Results from the Nelson-Denny Reading Test for Colleges and Senior High Schools, Form A.
3. Results from the Butler Test for Mathematical Concepts.
4. Results from the Kilzer-Kirby Inventory Test for the Mathematics Needed in High School Physics, Part I.

5. Results from the Carter Test for Mathematical Concepts in High School Physics.
6. The distribution of teachers' marks in physics for the first semester of 1930-31.

As indicated in Part III, the first three tests were used mainly for certain comparisons and analyses. Results from the last two tests were used in attempting to realize the major purposes of this study. A full consideration of certain more or less subsidiary phases of the problem necessitated the use of teachers' marks in physics.

As far as possible, interpretations of results from the series of tests are reserved for Part VII.

The Results from the Otis Self-Administering Tests of Mental Ability, Higher Examination, Form B

Since it was believed that any study of mathematical abilities in high school physics would be more significant in the light of certain comparisons with a measure of intelligence, the Otis Self-Administering Tests of Mental Ability, Higher Examination, Form B, were administered to every student included in this study. Certain brief references to the performance of the subjects on this test are made in Part IV in the description of the subjects employed in this study. A more complete presentation of the results of the intelligence examination is now necessary.

The distribution of intelligence quotients for the 404 subjects included in this study, according to the results from the Otis Self-Administering Tests of Mental Ability, Higher Examination, Form B, is fairly symmetrical with no very apparent skewness in either direction and no unusual piling up of scores at either end of the scale. The range is from 70 to 130, a difference of 60 points between the best and the poorest student. The median intelligence quotient is 105.9 and the mean is 105.3. Thus the mean and median differ by only six-tenths of a point. The middle 50 per cent are between 96.4 and 113.9. The S. D. was found to be 11.83. A summary of these results is found in Table VI.

As a further indication of the nature of this group, rather interesting comparisons may be made with Anderson's study of 237 sophomores, juniors, and seniors in the University of Missouri.[1] The best

[1] Anderson, E. M. *Individual Differences in the Reading Ability of College Students,* The University of Missouri Bulletin, Vol. 29, No. 39. 1928. p. 16.

TABLE VI

A Distribution of the Intelligence Quotients of 404 High School Physics Students According to the Otis Self-Administering Tests of Mental Ability, Higher Examination, Form B

Intelligence Quotient	Frequency
130–134	2
125–129	8
120–124	35
115–119	43
110–114	61
105–109	65
100–104	54
95– 99	49
90– 94	44
85– 89	28
80– 84	12
75– 79	2
70– 74	1
Number	404
Median	105.9
Q1	96.4
Q3	113.9
Range	70–130
Mean	105.26
S. D.	11.83

Table reads: On the Otis Self-Administering Tests of Mental Ability, two students were found to have intelligence quotients of 130 to 134, eight intelligence quotients of 125 to 129, etc.

student in the high school group has approximately the same intelligence quotient as the best student in Anderson's group, but the range is from 18 to 21 points greater for the high school group. The mean for the high school group is from seven to ten points lower and there is a higher S. D. for the high school group. A summary of these comparisons is found in Table VII.

TABLE VII

A Comparison of the Intelligence Quotients of 404 High School Physics Students and 237 Students in the University of Missouri

	H. S. Physics Students	University of Missouri Fall	Winter
Number	404	145	92
Range of I. Q.'s	70–130	88–130.5	91.5–129
Mean	105.26	115.8	112.23
S. D.	11.83	9.3	8.35

Since certain comparisons were to be made with scores on the intelligence test, a distribution was made on this basis.

The range of scores for all pupils on this test is 12 to 69, or 57. The average of the scores is 45.9 and the median is 46.7. The difference of eight-tenths of point between the mean and median furnishes some evidence of a fairly normal distribution. The median chronological age for the group was found to be seventeen years two months. According to the published norms on the Otis test the median score for this age is 41.0. The median for this group, therefore, is 5.7 score points above the norm for the median chronological age of seventeen years two months.

The middle 50 per cent are between 37.7 and 54.0. The standard deviation for the group is 11.18 and the coefficient of variation is 24.36. A summary of these facts is given in Table VIII.

Comparisons between the performance of boys and girls on the Otis test are also summarized in Table VIII. The median score for the 274 boys included in this study is 46.1 as compared to 48.0 for the 130 girls. These medians differ by 1.9 points in favor of the girls. The mean of the scores for the boys is 45.8 and for the girls it is 46.3, a difference of one-half point. The S. D. is approximately 1.3 points greater for the boys than for the girls. The coefficient of variation also shows the boys to be about 14 per cent more variable on this test than are the girls. These differences, with the possible exception of variability, are obviously not great enough to be very significant.

The Results from the Nelson-Denny Reading Test

There were three main purposes in giving the Nelson-Denny Reading Test in connection with this study. First, it was felt that it would be desirable to use reading test results in describing the general nature of the subjects employed in the study. Second, it was believed that results from a general reading test would be useful in describing the nature of the abilities measured by the Carter Test for Mathematical Concepts in High School Physics. In view of the fact that reading ability very obviously enters into performance on such a test as the Carter test, it was particularly necessary to make certain comparisons in terms of correlations with reading ability and with the abilities measured by other mathematics tests. Third, reading test results were needed for the purpose of determining the rela-

tive importance of various mathematical abilities with respect to performance in high school physics.

TABLE VIII

A Distribution of Scores of 404 High School Physics Students on the Otis Self-Administering Tests of Mental Ability, Higher Examination, Form B

Score	Boys	Girls	Total
69–72	1	0	1
65–68	8	1	9
61–64	25	6	31
57–60	20	11	31
53–56	21	18	39
49–52	34	24	58
45–48	38	20	58
41–44	26	12	38
37–40	37	9	46
33–36	25	12	37
29–32	17	9	26
25–28	12	5	17
21–24	8	2	10
17–20	1	1	2
13–16	0	0	0
9–12	1	0	1
Number	274	130	404
Median	46.1	48.0	46.7
Q1	37.5	38.6	37.7
Q3	54.2	53.8	54.0
Range	12–69	19–67	12–69
Mean	45.8	46.3	45.9
S. D.	11.58	10.27	11.18
Coefficient of variation	25.28	22.18	24.36

A brief reference to the results from the Nelson-Denny test is made in Part IV in connection with the description of the nature of the subjects employed in this study. It is shown there that the general performance of the subjects on the Nelson-Denny test was relatively high. It is now necessary to present more fully the results from the Nelson-Denny test since these results are the basis for certain comparisons discussed in Part VII. A summary of the results from the Nelson-Denny Reading Test is given in Table IX.

An inspection of Table IX shows a rather wide range of reading ability among the subjects included in this study. The range of scores is from 12 to 129 with a median of 70.5 and a mean of 70.4. The norm for high school seniors on this test is 57. The median for the group of 404 physics students, one-third of whom were juniors, is slightly above the seventy-fifth percentile for seniors according to the

authors' norms. The middle 50 per cent of the group are between 53 and 86.4.

According to the results from this test, the best student in the group reads almost twice as rapidly and accurately as does the median student. The coefficient of variation of 32.2 on this test is large enough to indicate a remarkable variability in performance.

A further reference to Table IX shows that the differences between boys and girls with respect to performance on the reading test are not great enough to be considered very significant, with the possible exception of variability as measured by the coefficient of variation. The boys are about 11.4 per cent more variable in reading ability than are the girls. The difference of about four points in the medians in favor of the girls is equivalent to a reaction to two more words and to about one-half of a paragraph more in a period of thirty min-tain conclusions which are contained in succeeding pages.

Results from the Butler Test for Mathematical Concepts

The chief purpose for using the Butler test in this study was to serve as a criterion for correlations and comparisons with the Carter test which was designed to measure similar abilities in physics situations. Results from the Butler test were also used in support of certain conclusions which are contained in succeeding chapters.

The Butler test was designed for use in the junior high school, and the distribution of scores for the 404 high school physics students included in this study indicates that it was somewhat easy for this group. The distribution of scores is skewed in the direction of the high end of the scale. This is the only test in the series upon which any student made a perfect score, two having made correct responses to all items. The range of scores for the group of high school physics students is from 27 to 63, or 36. This is the shortest range on any test in the series in proportion to the length of the test. The median score for the group is 50.3 which is nearly five-sixths of the perfect score of 63. The compactness of the distribution is also indicated by the standard deviation of 7.5 and the coefficient of variation of 15.3. The results from the Butler test are summarized in Table X.

An inspection of Table X shows that there are no particularly significant differences between boys and girls with respect to performance on the Butler test. The differences in the medians and the

means for the two groups are 2.7 points and 2.2 points, respectively, in favor of the boys. The difference in variability between boys and girls on this test is smaller than is the case with either the intelligence test or the reading test. On the Butler test, the boys are 1.3 per cent more variable than the girls, though it will be remembered that the boys are 14 per cent more variable than the girls on the intelligence

TABLE IX

A Distribution of Scores of 404 High School Physics Students on the Nelson-Denny Reading Test for Colleges and Senior High Schools

Score	Boys	Girls	Total
125–129.9	1	0	1
120–124.9	1	1	2
115–119.9	2	2	4
110–114.9	10	2	12
105–109.9	6	3	9
100–104.9	8	6	14
95– 99.9	12	4	16
90– 94.9	12	7	19
85– 89.9	21	12	33
80– 84.9	18	13	31
75– 79.9	25	9	34
70– 74.9	16	14	30
65– 69.9	21	11	32
60– 64.9	24	10	34
55– 59.9	13	5	18
50– 54.9	25	10	35
45– 49.9	20	4	24
40– 44.9	13	5	18
35– 39.9	11	7	18
30– 34.9	4	3	7
25– 29.9	5	2	7
20– 24.9	3	0	3
15– 19.9	2	0	2
10– 14.9	1	0	1
Number	274	130	404
Median	68.8	72.9	70.5
Q1	51.9	56.5	53.0
Q3	86.1	86.9	86.4
Range	12–129	27–124	12–129
Mean	69.3	72.2	70.4*
S. D.	23.3	21.3	22.7*
Coefficient of variation	33.6	29.5	32.2*

* From a distribution with a step interval of 10.

test and 11.4 per cent more variable than the girls on the reading test. Conclusions regarding differences in performance between boys and girls on the Butler test are somewhat less reliable than is the case with

the other tests because of the smaller number of items on the test and because of the fact that it was too easy for certain members of the group.

TABLE X

A Distribution of the Scores of 404 High School Physics Students on the Butler Test for Mathematical Concepts

Score	Boys	Girls	Total
61–63.9	11	0	11
58–60.9	22	6	28
55–57.9	41	15	56
52–54.9	46	19	65
49–51.9	53	20	73
46–48.9	27	20	47
43–45.9	19	19	38
40–42.9	16	12	28
37–39.9	20	7	27
34–36.9	11	7	18
31–33.9	4	2	6
28–30.9	3	2	5
25–27.9	1	1	2
Number	274	130	404
Median	51.0	48.3	50.3
Q1	45.1	43.2	44.2
Q3	55.4	53.2	54.7
Range	27–63	27–60	27–63
Mean	49.8	47.6	49.1
S. D.	7.6	7.2	7.5
Coefficient of variation	15.3	15.1	15.3

There is, however, a rather remarkable consistency between the results herein presented and those reported to the writer by the author of the Butler test. Butler[2] reports an average grade-to-grade gain of 6.8 points on this test for the junior high school grades. The median for the 404 physics students, approximately one-third of whom were juniors, is 10.4 higher than the median for ninth-grade pupils. Butler also reports that in the eighth and ninth grades, boys are slightly superior to girls. Table XI summarizes these comparisons.

Results from the Kilzer-Kirby Inventory Test for the Mathematics Needed in High School Physics

As indicated in Part III, the Kilzer-Kirby test was used in this study to measure computational abilities in physics situations. It

[2] Data for the Butler test for Grades VII to IX, inclusive, as presented in Table XI, were furnished by Dr. Charles H. Butler, the author of the test.

TABLE XI

A Comparison of the Dispersions of Scores on the Butler Test for Grades Seven, Eight, and Nine with Those for 404 High School Physics Students

Grade	VII	VIII	IX	404 Physics Students
Median	26.1	32.3	39.9	50.3
Range	46	53	54	36
S.D.	8.45	8.17	8.88	7.5
Difference in Md. Boys over Girls	-0.2	1.95	1.78	2.7

was also used in studying relationships between performance in computational situations and performance in situations involving the recognition of mathematical concepts. For these reasons, the results from the Kilzer-Kirby test are described in greater detail than are the results from the criterion tests which are described in the foregoing sections of Part VI.

As used in this study, the Kilzer-Kirby test may be considered as a power test since no time limit was set for it. Therefore, the results may be considered as typical of what high school physics students are really able to do on this test.

The median for the 404 students included in this study is 45.2 as compared to the median of 37 reported by the author in the manual accompanying the tests. The middle 50 per cent are between 37.2 and 53.6 as compared to the inter-quartile range extending from 30 to 44 reported by the author. Since Kilzer allowed only forty minutes for the administration of the test, it is entirely probable that these differences are due to the time allowed rather than to any difference between the two groups.

The mean performance on this test for the group included in this study is 44.3, or nine-tenths of a point lower than the median of 45.2. Since the highest possible score on this test is 66, these results indicate that the mean performance is about two-thirds of the possible score.

The wide distribution of scores on this test, ranging from 16 to 64, is of interest. This indicates that with respect to computational abilities, the poorest pupil is about one-fourth as efficient as the best pupil. A rather wide variability in performance on this test is also indicated by the coefficient of variation of 25.24. The facts regarding general performance on the Kilzer-Kirby test are summarized in Table XII.

Inspection of Table XII indicates that there may be some significant differences in performance on this test between boys and girls. The median for the 274 boys is 46.5 as compared to a median of

40.0 for the 130 girls. The coefficient of variation for the boys is 24.22 and for the girls it is 26.67. The girls are thus found to be about 10 per cent more variable than the boys as far as computational abilities are concerned. This difference is the more interesting and significant when it is remembered that the situation is reversed in the case of intelligence and reading ability. In intelligence, boys were found to be about 14 per cent more variable than girls, but the median score for the girls is 1.9 points higher. In reading, as measured by the Nelson-Denny test, the boys are about 11.4 per cent more variable than the girls, but the median for the girls is about 4 points higher than the median for the boys.

TABLE XII

A Distribution of the Scores of 404 High School Physics Students on the Kilzer-Kirby Inventory Test for the Mathematics Needed in High School Physics

Score	Boys	Girls	Total
64–66	1	0	1
61–63	9	3	12
58–60	28	11	39
55–57	29	5	34
52–54	22	11	33
49–51	29	7	36
46–48	23	16	39
43–45	23	6	29
40–42	25	6	31
37–39	27	25	52
34–36	12	6	18
31–33	14	9	23
28–30	6	9	15
25–27	10	8	18
22–24	9	3	12
19–21	5	3	8
16–18	2	2	4
Number	274	130	404
Median	46.5	40.0	45.2
Q1	38.2	33.5	37.2
Q3	54.8	50.9	53.6
Range	17–64	16–63	16–64
Mean	45.4	42.0	44.3
S. D.	11.1	11.2	11.2
C. V.	24.23	26.67	25.24

Thus far, results from the Kilzer-Kirby test have been presented as a whole. It has been shown that there is a wide variation in performance on this test but that the median for the group is about two-thirds of the highest possible score. This indicates a rather high performance for the group as a whole. This should be remembered in reading the

next few paragraphs, which deal with data concerning the performance on various items of the test.

The items on the Kilzer-Kirby Inventory Test for the Mathematics Needed in High School Physics, Part I, are supposed to be arranged in the order of difficulty, the easier ones appearing first.[3] In general the results from this test show that this is true for the group, even though the results do not correspond exactly to the order given in the test.

Sixteen items out of the sixty-six included in the Kilzer-Kirby test were missed by more than 50 per cent of the 404 pupils included in this study. Ninety-one per cent of the group failed to make the correct response to a problem involving the finding of a square root to three decimal places. Eighty-seven per cent of the group could not solve an equation involving the formula for converting centigrade temperatures into Fahrenheit. Similar facts may be read from Table XIII, which contains the text of each item from this test that was missed by 50 per cent or more of the group together with the number and percentage of students making incorrect responses to each of these items.

While it is not surprising to find that certain members of this group would make incorrect responses to the more difficult items on the Kilzer-Kirby test, it is rather remarkable that so many should fail on some of the easier and simpler items. A further analysis of the data from this test shows that approximately one-third to one-half of the group did not make correct reactions to such problems as the following:[4]

6 is what per cent of 30?

How many cu. in. are there in one cu. ft.?

Change .045 to per cent.

Robert has $100. His money is to John's as 4 is to 5. How much money has John?

Multiply —b by —b.

The diameter of a circle is 10 inches. Find the circumference.

It was found further that approximately one-fourth of the group missed such problems as the following:

[3] Kilzer, L. R., and Kirby, T. J., *Directions for an Inventory Test for the Mathematics Needed in High School Physics,* p. 2. Public School Publishing Company, 1929.

[4] See the Kilzer-Kirby test form. In the order quoted the items are Nos. 52, 47, 43, 41, 23, 53, 25, 21, 34, 42, and 30.

TABLE XIII

A Summary of the Performance of 404 High School Physics Students upon the More Difficult Items of the Kilzer-Kirby Test

Text of Item	No. Wrong Responses	Per Cent of Group
Find the sq. rt. of 356.048 to three decimal places.	369	91
$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ If p equals 18, and f equals 1.8, find q.	351	87
Find t^2 if $\frac{1}{2}at^2=2s$	311	77
How many cu. in. in 1 gal.?	310	77
$\frac{A}{B}=\sqrt{\frac{X}{Y}}$ If A equals 3, B equals 1, and Y equals 4, find X.	307	76
Find the reciprocal of 5.	302	75
Simplify $\frac{4}{7/2}$	272	67
$\frac{F-32}{180}=\frac{3}{100}$ Find F.	266	66
Item 36. (Setting up an equation)	259	64
The radius of a circle is 2 in. The area is	246	61
How many inches in one meter?	235	58
Expand: $(m-n)^2$	231	57
How many cm. are there in 1 in.?	227	56
Item 54. (Asking for formula used)	225	56
Change 1/9 to decimal form. Show three decimal places.	211	52
If $v=at$, find a.	202	50

Table reads: In the item on the Kilzer-Kirby test which involved finding the square root of 356.048, 369 pupils out of 404, or 91 per cent, made incorrect responses; etc.

Find the square root of 16/25.

The altitude of a rectangle is 2 ft. and the base is 6 ft. Find the area in sq. ft.

Find the value of $3/4\times8/9\times10/15$.

If 5% of a number is 10, find the number.

Find the square root of $49a^2b^4$.

In spite of the comparatively high central tendency of the group, it is rather disturbing to find that from one-fourth to one-third of the juniors and seniors in high school physics use their mathematics as poorly as these results indicate. The Kilzer-Kirby test results for

this group, however, tend to support certain conclusions of other investigators whose studies have been summarized in Part I. Evidence that this test is adequate for measuring in this group the abilities for which it is intended is found in the fact that no pupil made a perfect score, thus giving a complete distribution on the test, and in the fact that there is fairly wide range of performance on it.

Results from the Carter Test for Mathematical Concepts in High School Physics

As stated in Part III, the research test developed in connection with this study was designed to measure the student's ability to recognize the mathematical concepts involved in reading high school physics materials. The nature of this ability and that of the test are discussed in some detail in Part III. Evidence is also presented there to show that the test has a fair degree of reliability and objectivity. The test is also shown to be valid in that the test items were taken directly from the physics context.

(A) STATISTICAL EVIDENCE OF VALIDITY

One point with respect to the validity of the Carter test remained to be demonstrated after the returns were all in. If it measured what it purported to measure, it should show a higher correlation with other tests of mathematical ability than with intelligence or reading ability as usually measured by such tests as the Otis test and the Nelson-Denny test, which were the other criteria employed. The correlations between the research test results and results from the four other tests used in this study are given in Table XIV.

TABLE XIV

Correlations Between the Carter Test for Mathematical Concepts in High School Physics and Four Other Tests

	Butler	Kilzer-Kirby	Otis	Nelson-Denny
Carter	$.65 \pm .019$	$.66 \pm .019$	$.56 \pm .023$	$.55 \pm .023$

Table reads: The correlation between results on the Carter test and the Butler test is $.65 \pm .019$, etc.

The correlation between the results from the Carter test and those from the Butler test, which purports to measure the same general type of ability, is .09 higher than the correlation between the Carter test results and the Otis test results. The correlation between the re-

sults from the Carter test and those from the Kilzer-Kirby test, which is also a mathematics test, is .10 higher than the correlation between the Carter test results and the Otis test results.

By means of the formula for the probable error of the difference between two correlations and with the help of the appropriate probability table, it was established that there were 98 chances in 100 that the first difference of .09 (Carter-Butler vs. Carter-Otis) is a significant difference greater than zero.[5] In a like manner it was determined that there were 99 chances in 100 that the second difference of .10 (Carter-Kilzer-Kirby vs. Carter-Otis) is a significant difference greater than zero.

The correlation between the results from the Carter test and those from the Kilzer-Kirby test is .11 higher than the correlation between the Carter test results and the Nelson-Denny test results. The correlation between the Carter test results and the Butler test results is .10 higher than the correlation between the results from the Carter test and those from the Nelson-Denny test. It was found that there were 99 chances in 100 and 98 chances in 100, respectively, that these differences were significant differences greater than zero.

These figures may be interpreted as offering some evidence of the validity of the Carter test.

(B) THE DISTRIBUTION OF SCORES ON THE CARTER TEST

The results from the Carter test are considered to be highly satisfactory for the purposes of this study. The group of 404 subjects ranges from 22 to 65, no pupil having made a perfect score. The highest score is approximately three times as high as the lowest. A fair degree of variability on this test was found, as evidenced by the range of 43 points out of 66 and by the coefficient of variation of 17.14.

As is the case with the other tests, the median is relatively high on the scale between zero and the highest possible score. The median for the 404 cases is 50.6 and the mean is 48.9 with a standard deviation of 8.38. The middle 50 per cent are between 44.0 and 55.6. These results indicate a relatively high degree of ability to recognize the mathematical concepts which are included in the Carter test, and are in harmony with the comparatively high central tendencies of the

[5] Garrett, H. E., *Statistics in Psychology and Education*. Longmans, Green, 1926, pp. 170-172 and p. 135.

group on the intelligence test and on the reading test. The results from the Carter test are summarized in Table XV.

TABLE XV

A Distribution of the Scores of 404 High School Physics Students on the Carter Test for Mathematical Concepts in High School Physics

Score	Boys	Girls	Total
64–66	3	1	4
61–63	14	3	17
58–60	25	15	40
55–57	40	10	50
52–54	42	24	66
49–51	39	15	54
46–48	27	15	42
43–45	30	16	46
40–42	16	6	22
37–39	15	14	29
34–36	9	5	14
31–33	8	3	11
28–30	1	1	2
25–27	4	1	5
22–24	1	1	2
Number	274	130	404
Median	50.8	49.6	50.6
Q1	44.5	43.3	44.0
Q3	56.0	54.6	55.6
Range	22–64	22–65	22–65
Mean	49.8	48.6	49.4
S. D.	8.38	8.34	8.38
C. V.	16.83	17.14	16.96

An inspection of Table XV shows that the differences between boys and girls in performance on this test are small. There is a difference in the medians for the two groups of 1.2 points in favor of the boys with about the same difference in the respective quartile points and the means. The girls are about 1.8 per cent more variable than the boys. These differences are not great enough to be considered significant.

(c) THE RELATION BETWEEN MATHEMATICS TRAINING AND PERFORMANCE OF THE CARTER TEST

Certain facts regarding the past training in high school mathematics for the group of subjects included in this study are presented in Part IV. It is shown that the members of this group had had from two to nine semesters of mathematics training with a mean of 4.8 semesters. Theoretically, at least, there should be some relation be-

tween past training in mathematics and the ability to recognize mathematical concepts in physics situations. The facts regarding the relationship between these two factors are summarized in Table XVI.

TABLE XVI

The Relation Between Mathematics Training by Semesters and Performance on the Carter Test for Mathematical Concepts in High School Physics

Semesters Training in Math.	Performance on the Carter Test Mean	Median	Q1	Q3	Number	Per Cent
0					0	0
1					0	0
2	45.2	44.8	40.0	48.7	40	10.0
3	45.1	45.0	38.0	54.5	16	4.0
4	46.8	48.8	42.5	53.2	148	36.7
5	49.0	50.8	42.8	56.4	58	14.4
6	51.4	52.3	46.9	57.6	79	19.6
7	54.8	56.6	52.6	59.6	49	12.1
8	53.3	56.0	51.0	58.0	12	3.0
9	59.5	59.5			2	0.5
All	49.4	50.6	44.0	55.6	404	100.3

An examination of Table XVI shows that there is a small but consistent gain in performance on the Carter test with each additional semester of mathematics training, with the exception of a small decrease in the mean between the second and third semesters and a decrease in both the median and the mean between the seventh and eighth semesters. These apparent inconsistencies in the data are probably due to the small numbers of students who have had three and seven semesters, respectively, of mathematics training. Leaving out of consideration the two students who reported nine semesters of mathematics training, the medians on the Carter test vary from 44.8 for the group with two semesters of training to 56.0 for those with eight semesters of training. This represents an average gain in performance on the Carter test of approximately 3.7 points per year of mathematics training. This figure becomes 3.9 if, because of the small number of pupils with more than seven semesters of mathematics training, we combine the figures for the groups with seven, eight, and nine semesters of training.

These results indicate that mathematics training beyond Grade IX may function to some extent in increasing the student's ability to recognize certain mathematical concepts in high school physics.

(D) THE RELATION BETWEEN TRAINING IN PHYSICS AND PERFORMANCE ON THE CARTER TEST

The determination of the relation between training in physics and performance on the Carter test was found to be necessary for a complete description and interpretation of the results from the Carter test. For this reason it was necessary to make certain comparisons between the performance of the main group of subjects and that of a comparable group which was nearing the end of a year's work in physics. For this purpose the Carter test was administered in the latter part of December, 1930, to 96 physics students who were within about two weeks of the close of the second semester of physics training in the Cleveland High School of St. Louis. These students had previously been given the Otis Self-Administering Tests of Mental Ability, Higher Examination, Form B, in connection with this study and found to be slightly superior in ability, but fairly comparable to the group of 404 high school physics students included in this study. A comparison of the performance of these two groups is given in Table XVII.

TABLE XVII

The Influence of Physics Training Upon Performance on the Carter Test for Mathematical Concepts in Physics

Group	Number	Otis Test Results			Carter Test Results	
		Med.	Range	I.Q.	Med.	Range
Cleveland 9½ Mos. Tr.	96	50.2	28–67	109.8	51.4	21–64
Our Group 2½–4 Mos. Tr.	404	46.7	12–69	105.9	50.6	22–65
Differences		3.5		3.9	0.8	0

An examination of Table XVII shows that there is a difference in the median scores of the two groups on the intelligence test of 3.5 points in favor of the Cleveland High School group. The range of scores on this test for the 96 students in this group is 28 to 67, while the range for the 404 students included in this study is from 12 to 69. The median intelligence quotient for the Cleveland High School group is 109.8 as compared to a median of 105.9 for the 404 students included in this study. This is a difference of 3.9 points in favor of the St. Louis group.

The median score on the Carter Test for Mathematical Concepts

in High School Physics for the second semester students, who had had approximately nine and one-half months of training in physics, is 51.4 as compared with the median of 50.6 for the students included in this study, who had had from two and one-half to four months of training in physics. The range of scores on the Carter test for the St. Louis group is from 21 to 64 as compared with a range of 22 to 65 for the group here studied.

This difference between the two groups on the Carter test is not great enough to justify the conclusion that physics training contributes to any great extent to performance in the abilities measured by that test. This finding establishes a basis for a more reliable interpretation of the data from the Carter test since it was not administered at the same time to all the groups co-operating in the study.

(E) PERFORMANCE ON THE CARTER TEST BY ITEMS AND PAIRS OF CONCEPTS

In the foregoing paragraphs the results from the Carter test are presented as a whole. It has been shown that there was a fair amount of variation in performance on this test, but that the central tendency for the group was relatively high. Data with respect to performance on separate items of the test will now be presented.

In the construction and preliminary administration of this test, no attempt was made to arrange the items in the order of difficulty. As has been pointed out in Part III, the Carter Test for Mathematical Concepts in High School Physics consists of two parts, each containing the same thirty-three mathematical concepts in the same sequence. As also indicated in Part III, an attempt was made to obtain an approximate equality of length and difficulty in the two items involving a given mathematical concept. In order to give a more complete description of performance on the Carter test, tabulations were made to show the percentage of incorrect responses made to each member of each pair of concepts. The method of pairing concepts and the percentage of incorrect responses to each member of each pair of concepts are summarized in Table XVIII.

A reference to Table XVIII shows that this attempt to equalize the difficulty of the two test items involving a given mathematical concept was rather successful. For example, Items 1 and 34 were missed by 31 per cent and 26 per cent, respectively, of the 404 students in the group, while Items 2 and 35 were missed by 10 per cent

TABLE XVIII

Performance on the Carter Test by Items and Pairs of Concepts

Concept	Items No. A	Items No. B	Per Cent of Error A	Per Cent of Error B
Negative Number	1	34	31	26
Formula	2	35	10	4
Algebraic Factor	3	36	63	58
Inequality	4	37	18	31
Direct Proportion	5	38	39	32
Ratio	6	39	28	20
Constant	7	40	52	19
Indirect Measurement	8	41	19	15
Approx. in Meas.	9	42	10	7
Perpendicular	10	43	16	27
Parallel	11	44	47	49
Minimum	12	45	40	46
Square Root	13	46	18	8
Coefficient	14	47	43	28
Equation	15	48	31	26
Solution of an Equation	16	49	7	3
A Power	17	50	26	25
Exponent	18	51	59	44
Limit	19	52	58	8
Identity	20	53	23	20
Area	21	54	59	15
Volume	22	55	32	13
Dimensions	23	56	16	13
Maximum	24	57	5	13
Positive Number	25	58	24	28
Graphic Representation	26	59	36	19
Direction	27	60	25	16
Inverse Proportion	28	61	40	33
Variation	29	62	18	15
Algebraic Product	30	63	16	26
Algebraic Numbers	31	64	36	44
Equality	32	65	5	15
Dependence	33	66	8	8
Average Per Cent of Error Per Item			29.0	22.8

Table reads: The concept, *Negative Number,* occurring in Items 1 and 34 of the test, was missed by 31 per cent and 26 per cent, respectively, of the 404 subjects, etc.

and 4 per cent, respectively. The average percentage of error per item on the first half of the test was 29.0 as compared to an average percentage of error per item of 22.8 on the last half of the test.

A study of Table XVIII indicates that there is a considerable difference in difficulty between various mathematical concepts with respect to the student's ability to recognize them in the physics context. The difference in the degree of difficulty between the members

of a few of the pairs of concepts suggests that the ability to recognize a mathematical concept in its context in physics may be a function of something else than the mathematics involved. Certain conclusions regarding this aspect of the study are brought out in the succeeding pages of this chapter.

A comparison between performances on the easier and the more difficult pairs of concepts will throw some light upon the nature of the ability to recognize mathematical concepts in physics materials.

Two of the least difficult items on the Carter test include the concept of *dependence of one variable upon another* in the statements:

The speed of sound in air is found to increase with an increase in temperature.

The boiling point of an unconfined liquid must vary as the outside pressure varies.

These statements were included in Items 33 and 66, respectively, and each of these items was missed by eight per cent of the 404 subjects. It is probable that the high performance on these two test items is largely due to the students' familiarity with the concept of *dependence,* but it may also be partly due to the relative simplicity of the context. Among the more difficult items on the Carter test are the two items which include the concept of *parallel* in these statements:

The waves from the sun or any distant object are without any appreciable curvature when they strike a lens; that is, they are so-called plane waves.

To find graphically the resultant of two concurrent forces, (1) represent the concurrent forces, (2) construct upon them as sides a parallelogram. and (3) draw a diagonal from the point of application.

These statements are contained in Items 11 and 44 and were missed by 47 per cent and 49 per cent, respectively, of the group. While the difficulty of these items may be due partly to the nature of the concept involved, it is obvious that, in contrast to Items 33 and 66, the greater complexity of the context is an important factor in determining the difficulty of these latter items.

These facts may also be illustrated by examples of pairs of items containing the same concept, in which there were the greatest differences in performance in terms of percentage of error.

Examples of pairs of concepts in which the members differ widely

in difficulty are the two items which involve the concept of *limit* in these statements:

In levers of the third class the acting force is between the resisting force and the fulcrum. The mechanical advantage is then obviously less than 1.

When inclined planes are used as machines, the friction is small, so that the efficiency generally lies between 90 per cent and 100 per cent and in no case can it be more than 100 per cent.

Item No. 19 in the test, which includes the first of the foregoing statements, was missed by 58 per cent of the subjects; while Item No. 52, which includes the second statement, was missed by only 8 per cent. The choices of responses, both right and wrong, under these items are exactly the same. The only apparent differences in these items are in the complexity of statement and in the fact that the second includes a clause at the end re-emphasizing the idea of limit. It may be possible that these differences in the items are sufficient to explain the difference in difficulty.

The concept of *area* occurs in these statements:

A smooth block is $10 \times 8 \times 3$ in. Compare the distance which it will slide when given a certain initial velocity on smooth ice if resting first on a 10×8 face and then on a 10×3 face.

Calculate the number of tons of atmospheric force on the roof of an apartment house 50×100 ft.

In this case, Item No. 21, which includes the first statement, was missed by 59 per cent of the subjects, while Item No. 54, which includes the second statement, was missed by 15 per cent of the group. The difference of 44 in the percentages of error on these items probably cannot be explained entirely in terms of the difficulty of the concept of *area* alone. The first item may be more difficult because of its greater length and complexity.

The concept of *constant* occurs in these statements:

One degree of change in temperature on the centigrade scale is such a temperature change as will cause the pressure exerted by a confined volume of hydrogen to change by 1/273 of its pressure at the temperature of melting ice.

The melting point of ice is a perfectly fixed, definite temperature, above which the ice can never be raised so long as it remains ice, no matter how fast heat is applied to it.

These statements are included in Items 7 and 40 and were missed by 52 and 19 per cent, respectively, of the subjects. It is probable

that the difficulty of these items may be explained on the basis of a lack of familiarity with the concept of *constant* and the difference in difficulty may be due to the more obvious implication contained in the second statement.

The foregoing examples serve as suggestions as to the difficulties encountered by the pupil in the reaction to mathematical concepts in physics and also suggest certain possible causes for these difficulties. Lack of training in certain mathematical facts may also be a contributing factor in this respect. It is not within the province of this study to describe in detail this phase of the results from the Carter test.

This completes the discussion of the results from the various tests employed in this study. There now remains the description of the distribution of first semester marks in physics which will complete this part.

The Distribution of First Semester Marks in Physics

Since one of the major purposes of this study is to determine the relative importance of two types of mathematical abilities in the success of pupils in high school physics, it was necessary to obtain the first semester mark in physics for each of the subjects employed in this study. These marks were also used as described in the preceding section in verifying the validity of the research test.

In order to obtain uniform reports of marks, letters were sent to each of the teachers of physics in the co-operating schools describing the grading system used in the University of Missouri[6] and asking them to report grades on that basis using plus and minus to indicate the higher and lower pupils in each group. Previous conversations or correspondence with these teachers had indicated that this system or a very similar one was used in all schools but one. Apparently no trouble was experienced by the various teachers in reporting grades on this basis. By this means grades were obtained for the 404 subjects on a thirteen point scale which was satisfactory for the purposes of correlations and comparisons. The distribution of first semester marks in physics is given in Table XIX.

The data upon which Table XIX is based were used for the correlations and comparisons with marks which are reported in Part VII.

[6] University of Missouri Catalog, 1930-1931. The University of Missouri Bulletin, Vol. XXXII, No. 1, The University of Missouri, Columbia, Missouri, January, 1931. pp. 101-102.

TABLE XIX

A Thirteen-Step Distribution of First Semester Marks in High School Physics for the 404 Subjects Included in This Study

Marks	Boys		Girls		All	
	No.	Per Cent	No.	Per Cent	No.	Per Cent
E+	0	0	0	0	0	0
E	16	5.9	3	2.3	19	4.7
E—	4	1.5	5	3.8	9	2.2
S+	18	6.6	11	8.5	29	7.2
S	25	9.1	14	10.0	39	9.6
S—	17	6.2	11	8.5	28	7.0
M+	36	13.1	12	9.2	48	11.9
M	76	27.7	39	30.0	115	28.5
M—	19	7.0	17	13.0	36	9.0
I+	10	3.6	4	3.1	14	3.5
I	40	14.6	8	6.2	48	11.9
I—	7	2.6	2	1.5	9	2.2
F	6	2.2	4	3.1	10	2.5
Totals	274	100.1	130	100.0	404	100.3

TABLE XX

A Five-Step Distribution of First Semester Marks in High School Physics for the 404 Subjects Included in This Study

Marks	Boys		Girls		All	
	No.	Per Cent	No.	Per Cent	No.	Per Cent
E	20	7.4	8	6.1	28	6.9
S	60	21.9	36	27.8	96	23.9
M	131	47.8	68	52.2	199	49.4
I	57	20.8	14	10.8	71	17.6
F	6	2.2	4	3.1	10	2.5
Totals	274	100.1	130	100.0	404	100.3

The same facts are shown in a more condensed form in Table XX.

A reference to Table XX shows that the distribution of first semester marks in physics is fairly normal and that it is fairly consistent with the usual percentages expected in each grade interval. According to Table XX, 49.4 per cent of the whole group received a grade of M. This is very close to the expected fifty per cent of the total number of students. Twenty-three and nine-tenths per cent of the group received a grade of S, and 6.9 per cent a grade of E. This makes a total of 30.8 per cent who made grades above M as compared to 20.1 per cent who received inferior or failing grades.

A reference to Table XX shows that the girls received better grades, on the average, than the boys. Thirty-three and nine-tenths per cent of the girls and 29.3 per cent of the boys received grades of S or better, while 13.9 per cent of the girls and 23.0 per cent of the boys received inferior and failing grades. This may be interpreted as meaning that the girls are under no handicap as compared to the boys as far as grades in physics are concerned.

Considering the relatively high performance of the group of subjects on the various tests employed in this study, it is believed that this distribution of marks is about normal for the group and that it is satisfactory for the purposes of this study.

Organizations Affiliated with The National Council of Teachers of Mathematics

1. Chicago Men's Mathematics Club. F. W. Runge, Sec., Evanston Township High School, Evanston, Illinois.
2. Alpha Mu Omega, Peru, Nebraska. Arthur L. Hill, Sponsor.
3. Louisiana-Mississippi Branch. Mr. B. A. Summer, Lumberton, Mississippi.
4. Detroit Mathematics Club. E. J. Corrigan, Cleveland Intermediate School.
5. Columbus Mathematics Club. Miss Marie Gugle, Assistant Supt. of Schools.
6. Section 19, (Mathematics) of the New York Society for the Experimental Study of Education. Dr. W. D. Reeve, Teachers College, New York City.
7. Women's Mathematics Club of Chicago and Vicinity. Mrs. Elsie P. Johnson, High School, Oak Park, Illinois.
8. New York Association of Mathematics Teachers. Ellis Johnson, 145 Lincoln Road, Brooklyn, New York.
9. Huntington Council of Teachers of Mathematics. Miss Florence Oxley, 933 12th Avenue, Huntington, West Virginia.
10. Springfield Mathematics Club. Miss Gertrude Keller, 101 S. Boulevard, Springfield, Missouri.
11. Buffalo Mathematics Club. Miss Hallie Poole, 674 Richmond Street.
12. Minneapolis Mathematics Club. Miss Ruth Olson, 2222 Nicollet Avenue.
13. Mathematics Section, Western Convention District, Pennsylvania State Education Association. Dr. Elizabeth Cowley, 913 Arch Street, Pittsburgh, Pa.
14. Mathematics Teachers, Bay Section, California Teachers Association. Miss Blanche Du Bois, 1725 San Jose Avenue, Alameda, California.
15. Mathematics Section, Oklahoma Education Association. Prof. J. O. Hassler, University of Oklahoma, Norman, Oklahoma.
16. Cleveland Mathematics Club. Mr. A. Brown Miller, Fairmont Jr. High School, Cleveland, Ohio.
17. Kentucky Section, Mathematical Association of America. Prof. Arthur R. Fehn, Centre College, Danville, Kentucky.

Preliminary Report of the Committee on Individual Differences

By Raleigh Schorling
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The National Council of Teachers of Mathematics, at the Washington meeting, authorized the organization of a Committee on Individual Differences which was instructed to investigate the matters of ability grouping, differentiated curriculums, and the like. To avoid a useless and expensive duplication of effort, it seemed desirable that this assignment be assumed by an existing committee operating under the direction of the North Central Association of Colleges and Secondary Schools known as the Committee on Unit Courses in Mathematics for Students of Low Ability. It seemed that it would be much better from every angle to have the National Council and the North Central Association cooperate in this difficult but crucial investigation.

Importance of the problem.—There are many teachers who believe that the most important problem before the teachers of mathematics, and perhaps of most of the other subjects of the secondary school, is to find out what students of low ability can do with profit. The problem is getting to be more and more perplexing. We hear a great deal about education for a changing world. The world is changing; but that need not concern us especially, for it has always been changing. Any child in any community, in any period of history, has had to make a tremendous number of adjustments. One cannot read the stirring tales of pioneer days without realizing the enormous difficulties involved in the educational problems faced by our forefathers. The problems of health, of citizenship, of making a living, and of recreation were all extremely difficult. Today light, heat, power, and music come and go with the push of a button. We need not be concerned by the fact that the world is changing, but we do need to analyze the situation to see *how* it is changing. Now one of the most effective factors in the high school classroom situation is the new philosophy of education. This new philosophy has many splendid aspects which space does not permit us to discuss. The one phase

which greatly modifies the high school teacher's task is that one which causes us to promote children on bases other than mastery of subject matter. In the last decade the passing mark of sixty or seventy in subject matter became a myth. In this era the teacher in the progressive school fundamentally concerned with health and citizenship rejected the indefensible practice of keeping ten-, eleven-, and even twelve-year-old children in the primary grades solely because of the fact that they had not advanced beyond average reading ability in these grades. The immediate effect of this new philosophy of education is that it throws into the high school classes large groups of children who are low in the fundamental skills. For example, a principal administered an achievement test in reading to pupils enrolled in geometry and Latin and found that in a class of 36 pupils she had, as regards reading ability, 1 fourth grade pupil, 3 fifth graders, 4 sixth graders, 19 seventh graders, 3 eighth graders, 3 ninth graders, and 3 that were so high as to be off the scale. Some years ago a noted writer pointed out the existence of laggards in our schools. In that investigation a "laggard" was defined as a pupil who was over-age chronologically. It appears that laggards of this type are rapidly disappearing but they are showing up as laggards or as over-age pupils in the fundamental skills of the school subjects. Specifically, the high school teacher is expected to teach Latin and geometry to a considerable number of pupils whose ability in reading does not exceed the *average* ability of fourth, fifth, and sixth grade pupils.

Psychologists and psychiatrists point out the fact that the placement of individuals in situations where mastery and understanding are difficult frequently results in all sorts of queer protective measures. In short, we are here face to face with the problem of social adjustment. Nobody knows the bad effects of the fear of failure. Public citizens who are concerned with the rising tide of crime, or at any rate with the increasing number of inmates of our prisons, suggest that the secondary school is responsible so long as it fails to meet the needs of the vast number that now are unadjusted to the curriculums.

Whether these pupils should be in school is an academic question. The fact is that they are here in our classes. We cannot get them out of our school, and even if we could, they could not find positions. The sensible thing to do, then, is to design curriculum materials that will fit their needs. The only way to do this, I think, is through classroom investigation. It is futile for a committee or the author of a

textbook to say, "This looks like good material for slow children," when it turns out in fact that such pupils cannot learn it. The first task, then, is to get more light on what children at different levels of ability, particularly those in the lower levels, can do with profit under properly controlled conditions.

Working under the joint auspices of the National Council and the North Central Association our committee has undertaken this research problem. The present economic situation may make it more difficult to carry on research in the years just ahead of us. Funds for research are rapidly disappearing, not only from the budgets of city school systems, but also from the universities. On the other hand, the unemployment situation is forcing large numbers of unadjusted pupils back into the schools and compelling school people to do what they can to meet the situation. In Chicago an investigation of ability grouping has been undertaken for the schools by Miss Mabel Sykes. She visited a large number of schools in an effort to get light on this phase of the problem. Miss M. Cottell Gregory, supervisor of mathematics in the schools of Louisville, Kentucky, was instructed by the Board to spend a year of her time in the pursuit of this problem. Indianapolis is reorganizing its course of study in mathematics with the hope that one of the chief outcomes will be an organization of materials and methods adjusted to the low group. Since 1929 Cleveland has had three courses of study for three levels of ability. Under the direction of Wm. L. Connor, Director of Research, this city has been trying to make adequate provision for "Z" pupils. At the Washington meeting of our committee representatives of many cities reported efforts to outline a definite program for the unadjusted pupils. Perhaps no better evidence of interest is available than the fact that the committee which appears in these pages is made up of people in key positions who are especially interested in and concerned with, this problem.

Method of attack.—The outline of the procedures to be employed can only be tentative. In fact, the chief purpose of this preliminary report is to invite the assistance of all who are interested in this investigation. We need suggestions as to the best ways for the committee to proceed. Naturally the details will need to be worked out. Mimeographed materials that have been used with slow groups will be helpful and, most of all, the committee needs to have careful study of pupil responses to a given unit of subject matter. How much did the pupils know when they started the work? What did they know

and what were they able to do when they had completed the unit? Did the pupils enjoy the work? These are some of the questions that we should like to answer not solely on the basis of teacher opinion, valuable as that is, but on the basis of a systematic record. Realizing that we have made scarcely a beginning on this type of investigation, and being well aware of the obstacles that confront us as regards the the lack of economic support for investigations of this type, the committee cannot confidently predict rapid progress or especially valuable outcomes. The committee can only promise to do what it can and believes that the great interest in the problem, and its importance, will result in something worth while.

Guides for the writing of a unit.—Our experience with material sent for the consideration of the committee suggests that we need to have common understandings about some points that can be used as guides in the writing of a unit.

(1) The unit should *not* be designated as a part of any specified grade. For example, the phrase "seventh grade" should not appear anywhere in the manuscript. The fact is that we do not know enough about the grade placement of materials for slow pupils. One teacher may wish to use a unit in the seventh grade whereas a second teacher in another city may decide to use the same unit in the ninth grade.

(2) The unit should be built from the ground up, specifically for slow groups and without any textbook references. Our committee is interested in *research* and not at all in the distribution of study guides for a particular textbook.

(3) Each unit should ignore the conventional *order* of topics. For example, a unit on percentage (or one perhaps on geometric constructions, the formula, bar and circle graphs, the equation) should not assume skills and information beyond the barest essentials ordinarily taught in the first six grades of arithmetic.

(4) A unit should include a supplement in which two things should appear: (*a*) a specific list of objectives on which mastery is to be achieved—this is intended for guidance to the teacher; and (*b*) a carefully constructed test covering the unit. It is intended that this test will be administered to the pupils who have done the unit.

Selection of units for investigation.—Much material has been received, but most of it is not suitable for extensive classroom trial. In fact, only one unit appears to have met the criteria listed in the preceding section. The experimental trial of this unit with an adequate number of pupils has been arranged. It is hoped that a pre-

liminary report will be available at the time of the Minneapolis meeting. To make doubly certain that the committee will be cautious in selecting only units of work that are likely to prove useful, two small subcommittees have been appointed to act as juries. One consists of three members near Ann Arbor, Michigan, and the other committee is made up of three members in or near Chicago, with Dr. E. R. Breslich as chairman. No unit will be recommended for experimental work until at least two members of each jury consider it promising. It is hoped that having these committees consist of small numbers of people near each other will help to keep expenses for travel and postage at a minimum. We may later need another subcommittee on the Pacific Coast for the selection of units.

A second sub-committee on publication needs to concern itself with mimeographing or printing the materials so as to make the units available for classroom trials. With research funds rapidly disappearing from various budgets this phase of the problem threatens to be extraordinarily difficult. It has been suggested that this committee consist of those who are willing to contribute $25 to the support of the investigation. It is hoped that in time the principal, together with reasonable interest, might be returned; but certainly no guarantee to this effect could be given under existing conditions. Already two teachers of mathematics have volunteered to serve on such a subcommittee. The chairman would be gratified to hear from other teachers of mathematics whose interest in the problem would be sufficiently great to cause them to volunteer not only their services but financial support to the investigation.

The Minneapolis meeting.—The next meeting of the National Council of Teachers of Mathematics is to be at Minneapolis in February, 1933. It has been suggested that the problem of individual differences and of related questions might well occupy the entire time of one session. If this program is approved by the directors of the Council, suggestions from readers of THE MATHEMATICS TEACHER concerning topics and investigations dealing with this topic will be appreciated by the committee responsible for this part of the Minneapolis program.

Membership of the committee.—A tentative list of members of the committee is given below. It is obvious that the committee is a strong representative group. The list includes many teachers who are willing to make a systematic and thorough study of this important problem. Moreover, some have unusual opportunities for investigations. The

size of the committee made it necessary to omit many mathematics teachers who have expressed great interest in the problem. However, all are urged to continue their interest and to contribute units for experimental trial. If any material is accepted by the committee, the person responsible for the unit will probably be added to the committee. Suggestions for changes in membership will be appreciated.

TENTATIVE LIST OF MEMBERS OF THE COMMITTEE ON UNIT COURSES IN MATHEMATICS FOR STUDENTS OF LOW ABILITY

Bell, Kate. Head of Mathematics Department, Lewis and Clark High School, Spokane, Washington.

Breslich, E. R. Professor of the Teaching of Mathematics, University of Chicago, Chicago, Illinois.

Burns, Katherine F. Teacher of Junior High School Mathematics, 4 Birch Road, Yonkers, New York.

Clark, Randolph. Chairman, Department of Mathematics, DeWitt Clinton High School, Mosholu Parkway and Gaynor Avenue, New York City.

Connor, Wm. L. Director of Research, Board of Education, Cleveland, Ohio.

Crawford, Mildred. Critic Teacher, Roosevelt High School, Michigan State Normal College, Ypsilanti, Michigan.

Dady, Margaret. Head of Mathematics Department and Assistant Principal, Waukegan Township High School, Waukegan, Illinois.

Engels, Bernice. Director of Mathematics and Commerce, Public Schools, Gary, Indiana.

Everett, John P. Professor of Mathematics, Western State Teachers College, Kalamazoo, Michigan.

Ewing, W. F. Asst. Superintendent of Schools, Oakland, California.

Fiedler, Adelaide L. Head of Mathematics Department, South Side High School, Fort Wayne, Indiana.

Forno, Dora M. New Orleans Normal School, New Orleans, Louisiana.

Gibson, Mrs. Cassie D. Critic Teacher, Central State Teachers College, Mount Pleasant, Michigan.

Gregory, M. Cottell. Chairman of Curriculum Revision Committee on Mathematics, Board of Education, Louisville, Kentucky.

Grogan, Ione H. Head of Mathematics Dept., Senior High School, Greensboro, North Carolina.

Hoge, James, Supervisor of Mathematics, University High School, Oakland, California.

Holley, Carmille. Assistant Professor of Mathematics, School of Education, Miami University, Oxford, Ohio.

Holmes, J. R. Superintendent of Schools, Muskogee, Oklahoma.

Jensen, George C. Assistant Superintendent of Schools, Sacramento, California.

Kanies, Mrs. Elizabeth. Supervisor of Mathematics, Chicago, Illinois.

Kemmerer, W. W. Director of Research, 617 Grt. Southern Building, Houston, Texas.

Klein, Carolyn. Teacher of Mathematics, Morey Junior High School, Denver, Colorado.

Kraybill, Dr. D. B. Superintendent of Schools, Wheeling, West Virginia.

Leonard, C. J. Head of Mathematics Department, Southeastern High School, 3030 Fairview Avenue, Detroit, Michigan.

Loss, Nellie. Head of Mathematics Department, Oak Grove Club, Flint, Michigan.

Loughren, Amanda. Supervisor of Mathematics, Jefferson High School, Elizabeth, New Jersey.

Miller, Helen R. Director of Mathematics, Hamtramck, Michigan.

Perdue, Dan H. State Supervisor of High Schools, Charleston, West Virginia.

Potter, Mary A. Supervisor of Mathematics, Racine, Wisconsin.

Robertson, Georgia D. Critic Teacher, Junior High School Mathematics, Morehead State Teachers College, Morehead, Kentucky.

Sauer, Florence M. Teacher of Mathematics, Wilbur Wright Junior High School, Dayton, Ohio.

Schorling, Raleigh. Head of Department of Mathematics, University High School, University of Michigan, Ann Arbor, Michigan.

Schreiber, Edwin W. Associate Professor of Mathematics, Western State Teachers College, Macomb, Illinois.

Shea, J. T. Assistant Director of Education, San Antonio, Texas.

Stirwalt, Ernest. Head of Mathematics Department, Herbert Hoover High School, Glendale, California.

Swineford, Helen, Withrow High School, Cincinnati, Ohio.

Taylor, Dr. S. Helen, University High School, Urbana, Illinois.

Taylor, Paul L. Head of Mathematics Department, Oakwood School, Poughkeepsie, New York.

Thiele, C. L. Director of Exact Sciences, Board of Education, Barlum Tower, Detroit, Michigan.

Touton, Frank C. Vice-President, University of Southern California, Los Angeles, California.

Whitney, Anna M. Head of Mathematics Department, Senior High School, Yakima, Washington.

Wilcox, Charles C. Assistant Supt. of Schools, Kalamazoo, Michigan.

Williams, Julia. Supervisor of Mathematics, Dearborn, Michigan.

Preliminary Report of the Committee on Geometry

By RALPH BEATLEY, *Harvard University*

AT THE MEETING of the National Council in Washington, February 20, 1932 the following motion was passed without dissenting vote: "The National Council of Teachers of Mathematics reaffirm their interest in the question of geometry as expressed at the annual meeting in Detroit, and hope that the Board of Directors will find it possible to initiate and carry forward a study of the whole question of geometry in our schools." The directors forthwith empowered the new president, Mr. Betz, to appoint a Committee on Geometry; and he, anxious that some work should go forward during the long time necessary for the final organization of this and other committees, selected six members of the National Council for immediate service on the committee, intending to enlarge the membership as soon as possible in order that it might include an even wider range of interest and opinion.

These first six members were Miss Gertrude E. Allen, formerly of Oakland, California; Professor Ralph Beatley of the Harvard Graduate School of Education; Miss Martha Hildebrandt, Maywood, Illinois; Mr. Joseph P. McCormack, New York City; Professor Vera Sanford of the School of Education, Western Reserve University; and Mr. Albert J. Schwartz of St. Louis. These six agreed to the following plan of action and set to work on the first item,which is purely preparatory in nature and not likely to cause embarrassment to those who might join the committee later.

The first item of the committee's plan of action was to assemble the suggestions which have been made since 1900, or thereabouts, concerning the teaching of geometry, both intuitive and demonstrative. These are to be found in THE MATHEMATICS TEACHER, the yearbooks of the National Council, the reports of committees, in doctorate dissertations, books on methods by American and foreign writers, and school geometries used here and abroad. The Committee is interested to garner ideas on the philosophy of geometry, and on its psychological aspects; to see also how these are reflected in the

choice of content and method. So far as possible it wishes to profit by the work of its predecessors. These various suggestions are being recorded in the form of bibliographical notes, mostly quite brief, but occasionally quite at length when the matter is novel and unfamiliar. To date we have reviewed most of the English, French, German, and Italian titles on our list and a considerable number of the American titles. This phase of the committee's work will be completed shortly.

Unwilling to believe itself omniscient and infallible, and mindful of the numbers of teachers of mathematics in this country who have valuable ideas on geometry which might easily be overlooked, the committee proposed as the second item of its plan the publication of a brief provocative article on geometry in THE MATHEMATICS TEACHER, this article to be designed especially to elicit comment from teachers of mathematics in general and from organized groups of teachers.

With this body of opinion before it the committee plans to proceed to the third item on its list, the detailed formulation of a few distinct philosophies of the teaching of geometry. Long lists of valid objectives in the teaching of geometry necessarily include some which conflict with one another. Quite different courses in geometry result from the different emphases accorded these conflicting items, the choice of emphasis in a given case depending upon the philosophy guiding him who frames the course.

The devising of a suitable procedure for the experimental trial of each philosophy thus formulated, probably with the assistance of expert counsel, constitutes the fourth item of the committee's plan. Co-operation with certain schools in experiments originating either in this committee or in the schools, and a final report collating the results of these various experiments, complete the committee's present plan of action. This plan is not to be considered as final, and will probably be modified as a result of suggestions from the enlarged committee. Advice and criticism from any source will be welcome at this time.

John Wallis

Born at Ashford, Kent, November 23, 1616
Died at Oxford, October 28, 1703

"It was always my affectation even from a child, not only to learn by rote, but to know the grounds or reasons of what I learn; to inform my judgment as well as furnish my memory."

THIS WAS Wallis's comment on a memorandum of his attainments in 1630 when he later claimed that he was fully prepared for the university although he did not actually begin his work there for another two years. He reports that at that time he both wrote and spoke Latin, and read Greek, Hebrew, and French. His training like that of other mathematicians of the period was in medicine and in theology, and it was not until he had taken his degree at Cambridge and had been ordained to the priesthood that he gave serious attention to mathematics as a career. It should be remembered that many university appointments at that date could be given only to the clergy. The memorandum mentioned above states also that he became acquainted with arithmetic in 1631, the subject then being considered as a "mechanical not an academic pursuit" and he studied it as a "pleasing diversion in spare hours."

From 1632 when he entered Emmanuel College until 1640, he was at Cambridge. On receiving his master's degree and being ordained, he acted as chaplain to a titled family for a short time and then held various appointments to churches in London. Here he became acquainted with Robert Boyle and with others who were interested in the new experimental philosophy and at this time he seems to have first thought of making mathematics his career. He seems to have been influenced in this decision by reading Oughtred's *Clavis mathematicae* and Torricelli's work on indivisibles. Boyle and the others constituted an association somewhat similar to Father Mersenne's group in Paris for the sharing of matters of scientific interest. In the 1650's they met in Oxford where Wallis had become Savilian Professor of Geometry. In the 1660's, the meeting place

was shifted to London, the king's patronage was secured, and the group became the Royal Society, Wallis being president during the later years of his life.*

In 1642, the Civil War being then in progress, Wallis astonished his patrons by reading a letter in code which told about the capture of a certain city. During the remainder of the war, his services were utilized by the Parliamentary party, but his skill in deciphering codes enabled him to preserve a somewhat independent position. Whether he actually with-held material from the Parliamentarians or whether the royalists only surmised that he did so, the facts are these: in 1648 he signed a remonstrance against the execution of Charles I; the following year Cromwell had him appointed as Savilian Professor at Oxford; in 1660 Charles II confirmed this appointment and shortly after he honored Wallis by appointing him on a commission for the revision of the Prayer Book. Details of Wallis's work with the various ciphers may be found in Ball's *Mathematical Recreations and Essays.*

Wallis corresponded with Fermat, submitted papers in Pascal's contest on the cycloid, studied Descartes's *Geometry* and himself produced a work with the title *De Sectionibus conicis* in which he used the symbol ∞ for infinity and in which he treated the curves as equations of the second degree and not merely as the sections of a cone. He wrote on the impact of inelastic bodies, invented a way to teach deaf-mutes to speak, acted as decipherer for William III, and in 1692 when consulted about the proposed adoption of the Gregorian calendar, advised against it on the ground that adopting it would imply submission to Rome.

His *Arithmetica Infinitorum* (Oxford, 1655) has been characterized as "the most stimulating mathematical work so far published in England." This contained work foreshadowing the integral calculus in finding areas under curves of the form $y=x^n$. Fermat and others had approached this problem, but Wallis's contribution seems to have been by dividing the abscissa into equal parts so that the difference between the inscribed and circumscribed rectangles erected on these segments was equal to the largest of these rectangles. Nunn's *Teaching of Algebra* should be consulted for Wallis's scheme. It is also described in Cajori's *History of Mathematics.* Interpolation to find the area of a circle led to the following value, the fraction $4/\pi$

* See Martha Ornstein, *The Rôle of Scientific Societies in the Seventeenth Century.*

being designated by a small square indicating that the value was that of the ratio of the square on the diameter to the area of the circle:

$$\frac{4}{\pi} = \frac{3 \times 3 \times 5 \times 5 \times 7 \times 7 \times 9 \times \&c}{2 \times 4 \times 4 \times 6 \times 6 \times 8 \times 8 \times \&c}$$

Wallis was able to find the values of the areas under curves of the form $y=(1-x^2)^n$ where n was a positive integer, but he was baffled by the problem where n was fractional. His work in this direction stimulated Newton to attempt the same problem. The *Source Book in Mathematics* contains a translation of Newton's work with this which led him eventually to the discovery of the Binomial Theorem for negative and fractional exponents, the work first appearing in print in Wallis's *Algebra* in 1685.

Wallis's *Mathesis universalis* (1658) was a more elementary treatise. His *De Algebra Tractatus; Historicus & Practicus* (1673; English translation 1685) contained an introduction dealing with the history of mathematics. It is noteworthy also for the treatment of imaginary quantities. He first shows that the square root of a negative quantity is a logical possibility and then shows that it has a geometrical counterpart in the mean proportional between two quantities one of which is positive and the other negative.†

Wallis's *Collected Works* (1693-99) contain Newton's account of fluxions and it should be remembered that Wallis as president of the Royal Society was able to smooth over the first questions at issue in the Newton-Leibniz controversy over the priority of the invention of the calculus, but that when the question was again raised it was, as the *Transactions of the Royal Society* note, after the death of Wallis and "the other old men" who had been in close touch with it.

VERA SANFORD

† See *Source Book in Mathematics*, p. 46.

A Tentative Program for the Policy Committee

By J. O. Hassler
University of Oklahoma

This program was outlined first by Mr. Betz, president of the National Council, and revised by the writer who has been appointed chairman of the committee.

In a letter written by Mr. Betz, we have an excellent summary of the proposed work of the Committee. "The average young teacher of mathematics starts with practically no training and without a definite vision as to the type of contribution that mathematics aims to make in the life of the pupil. It seems to me, therefore, that the primary task of a Policy Committee is to work out a platform or a set of guiding principles for the orientation and inspiration of the new arrivals in our profession.

"Even the more experienced teachers very often feel more or less vaguely, or without coherence, precisely what they are to do. They, too, need all the help we can give them by way of encouragement, stimulation, and suggestions. They would be helped tremendously, for example, by a representative set of references to the worthwhile literature on the subject, and perhaps by a compilation of significant quotations."

It is suggested that the separate parts of the Committee's job might be a consideration of the following:

1. The contribution of mathematics to civilization, past and present, coupled with an analysis of the prevailing narrow concept of mathematics as a tool subject, where only mechanical skills are learned.
2. A re-statement of the principal recommendations of the National Committee.
3. A formulation of a modern "credo" for the classroom teacher of mathematics.
4. Bringing up to date the study of the problem of transfer of training and formulating in clear and concise statements the present point of view as was done in the report of the National Committee.
5. A brief statement concerning the most worthwhile classroom procedures in mathematics.
6. A set of suggestions with reference to ways and means of arous-

ing the interest of our pupils in the study of mathematics, including suggestions concerning the organization of mathematics clubs.

7. Suggestions concerning a mathematical library and concerning the collection of helpful sources of material.

8. Suggestions concerning the future growth of teachers, with concrete illustrations of the practical teaching values obtained by a study of higher mathematics.

9. A new evaluation of the general objectives in teaching mathematics.

10. An examination of the modern educational theory of objective tests (and other new-type tests) with the purpose of determining the strength and weakness of these tests in measuring the most important results to be obtained by good mathematics teaching.

The formation of the committee is in progress and the list of members will be made public as soon as possible.

OFFICIAL NOTICE

As Secretary of The National Council of Teachers of Mathematics, I officially announce the annual election of certain officers of The National Council, said election to take place at Minneapolis, Minnesota, on Friday, February 24, 1933.* Article III, Section 7 of the by-laws states: "At least two months before the date of the annual meeting, all members shall be given the opportunity through announcement in the official journal to suggest by mail for the guidance of the Directors a candidate for each elective office for the ensuing year. At least one month before the annual meeting the Secretary of the Board of Directors shall send to each member an official ballot giving the name of two candidates for each office to be filled. These candidates shall be selected by a nominating committee of the Board of which the Secretary shall be chairman. The election shall be by mail or in person and shall close on the date of the annual meeting."

At the Detroit meeting, 1931, of The National Council, the nominating committee, consisting of the two most recent ex-presidents and the Secretary as chairman (for this year: Harry C. Barber, John P. Everett, and Edwin W. Schreiber), was instructed to prepare a primary ballot suggesting five eligible candidates for each elective office. The officers to be elected at the Minneapolis meeting are: (1) 2nd Vice-President 1933-1935; (2) Three Directors 1933-1936.

The periods of service of the officers of the National Council, from its organization in 1920 to the present time, are printed below.

(Signed) Edwin W. Schreiber
Secretary

* The dates for the next meeting of the National Council of Teachers of Mathematics was inadvertently given as February 17 and 18, 1932 in the October number of The Mathematics Teacher. The dates for the meeting are February 24 and 25. The headquarters are at the Hotel Nicollet in Minneapolis, as previously stated.

OFFICIAL PRIMARY BALLOT

For the Election of Officers at the February 24, 1933, Meeting of the National Council of Teachers of Mathematics

For Second Vice-President, 1933-35 (Vote for Two)

( ) Beatley, Ralph
Harvard University

( ) Miller A. Brown
Cleveland, Ohio

( ) Orleans, Joseph B.
New York City

( ) Rorer, Jonathan T.
Philadelphia, Pa.

( ) Stokes, C. N.
Philadelphia, Pa.

For Members of the Board of Directors, 1933-36 (Vote for Six)

( ) Barber, Harry C.
Boston, Mass.

( ) Cowley, Elizabeth B.
Pittsburgh, Pa.

( ) Croft, Mary E.
Buffalo, N.Y.

( ) Hassler, J. O.
Norman, Okla.

( ) Kee, Olive A.
Boston, Mass.

( ) Kelly, Mary
Wichita, Kan.

( ) Marquand, Helen H.
Columbus, Ohio

( ) Overman, J. R.
Bowling Green, Ohio

( ) Philips, A. W.
Emporia, Kan.

( ) Rankin, W. W.
Durham, N.C.

( ) Schlauch, W. S.
New York City

( ) Shriner, Walter O.
Terre Haute, Ind.

( ) Simpson, Helen
Lorain, Ohio

( ) Wallis, William
Washington, D.C.

( ) Wilson, Mildred
Evanston, Ill.

Please mark this ballot at once and mail same to Edwin W. Schreiber, Secretary, 619 West Adams Street, Macomb, Illinois. Kindly put your name and address on the outside of the envelope. If you prefer to make a copy of this ballot on a separate sheet of paper it will be acceptable.

THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Organized 1920—Incorporated 1928

PERIODS OF SERVICE OF THE OFFICERS OF THE NATIONAL COUNCIL

PRESIDENTS

C. M. Austin, Oak Park, Ill., 1920
J. H. Minnick, Philadelphia, Pa., 1921-1923
Raleigh Schorling, Ann Arbor, Mich., 1924-1925
Marie Gugle, Columbus, Ohio, 1926-27
Harry C. Barber, Exeter, N.H., 1928-1929
John P. Everett, Kalamazoo, Mich., 1930-1931
William Betz, Rochester, N.Y., 1932-33

VICE-PRESIDENTS

H. O. Rugg, New York City, 1920
E. H. Taylor, Charleston, Ill., 1921
Eula Weeks, St. Louis, Mo., 1922
Mabel Sykes, Chicago, Ill., 1923
Florence Bixby, Milwaukee, Wis., 1924
Winnie Daley, New Orleans, La., 1925
W. W. Hart, Madison, Wis., 1926
C. M. Austin, Oak Park, Ill., 1927-1928
Mary S. Sabin, Denver, Colo., 1928-1929
Hallie S. Poole, Buffalo, N.Y., 1929-1930
W. S. Schlauch, New York City, 1930-1931
Martha Hildebrandt, Maywood, Ill., 1931-1932
Mary A. Potter, Racine, Wis., 1932-33

SECRETARY-TREASURERS

J. A. Foberg, Chicago, Ill., 1920-1922, 1923-1926, 1927, 1928 (Appointed by Board of Directors)
Edwin W. Schreiber, Ann Arbor, Mich., and Macomb, Ill., 1929— (Appointed by the Board of Directors)

COMMITTEE ON OFFICIAL JOURNAL

John R. Clark, Editor, 1921-1928
W. D. Reeve, Editor, 1928—
Vera Sanford, 1929—
H. E. Slaught, 1928—

DIRECTORS

Marie Gugle, Columbus, Ohio, 1920-1922, 1928-1930, 1931-1933
Jonathan Rorer, Philadelphia, Pa., 1920-1922
Harry Wheeler, Worcester, Mass., 1920-1921
W. A. Austin, Fresno, Calif., 1920-1921
W. D. Reeve, Minneapolis, Minn., 1920, 1926-1927
W. D. Beck, Iowa City, Iowa, 1920
Orpha Worden, Detroit, Mich., 1921-1923, 1924-1927
C. M. Austin, Oak Park, Ill. 1921-1923, 1924-1927, 1930-1932
Gertrude Allen, Oakland, Calif., 1922-1924
W. W. Rankin, Durham, N.C., 1922-1924
Eula Weeks, St. Louis, Mo., 1923-1925
W. C. Eells, Walla Walla, Wash., 1923-1925
Harry English, Washington, D.C., 1925-1927, 1928-1930
Harry C. Barber, Boston, Mass., 1925-1927, 1930-1932
Frank C. Touton, Los Angeles, Calif., 1926-1928
Vera Sanford, New York City, 1927-28
William Betz, Rochester, N.Y., 1927-1929, 1930-1931
Walter F. Downey, Boston, Mass., 1928-1929
Edwin W. Schreiber, Ann Arbor, Mich., 1928-1929
Elizabeth Dice, Dallas, Tex., 1928, 1929-1931
J. O. Hassler, Norman, Okla., 1928, 1929-1931
John R. Clark, New York City, 1929-31
Mary S. Sabin, Denver, Colo., 1929-1930, 1931-1933
J. A. Foberg, California, Pa., 1929-
C. Louis Thiele, Detroit, Mich., 1931-33
John P. Everett, Kalamazoo, Mich., 1932-1934
Elsie P. Johnson, Oak Park, Ill., 1932-34
Raleigh Schorling, Ann Arbor, Mich., 1932-1934

New Members!

Some of our educational magazines have had to suspend publication because of a lack of support of the people these magazines are intended to serve. I hope this will never be the case with THE MATHEMATICS TEACHER.

However, from reports that come to this office, it is clear that at some of the state meetings this year nothing has been done by anyone at the mathematics sections of such associations to help us get new subscribers. Surely someone should always be delegated at such meetings to say a good word for the magazine and the yearbooks and also to distribute subscription blanks which can always be had from the office of THE MATHEMATICS TEACHER (525 W. 120th Street, New York City) for the asking.—THE EDITOR.